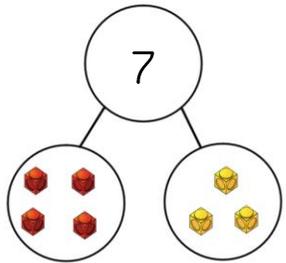
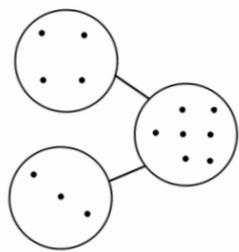


Part-Whole Model



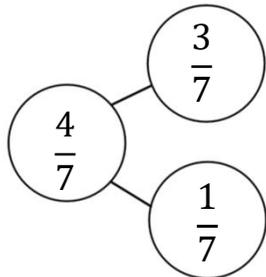
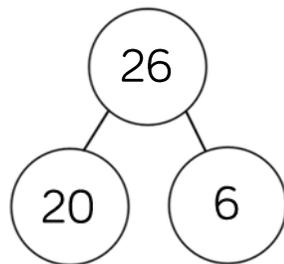
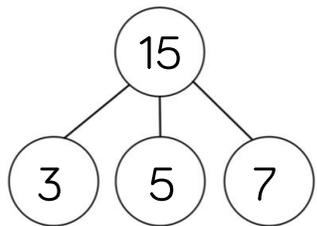
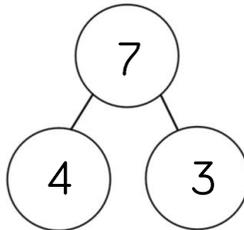
$$7 = 4 + 3$$

$$7 = 3 + 4$$



$$7 - 3 = 4$$

$$7 - 4 = 3$$



Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

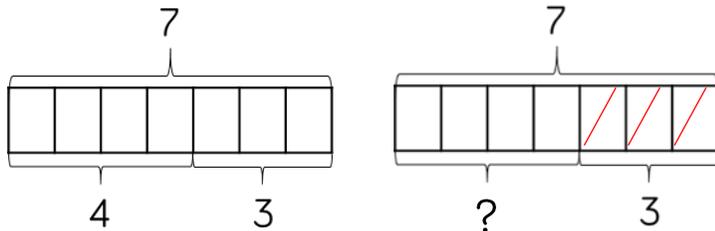
In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

Bar Model (single)

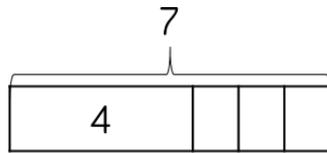
Concrete



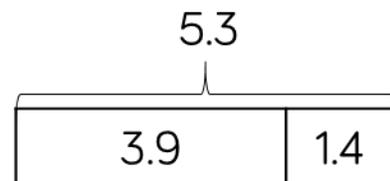
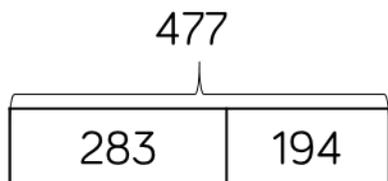
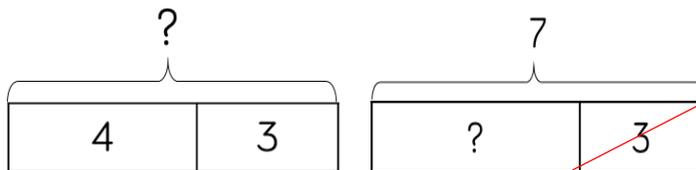
Discrete



Combination



Continuous



Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

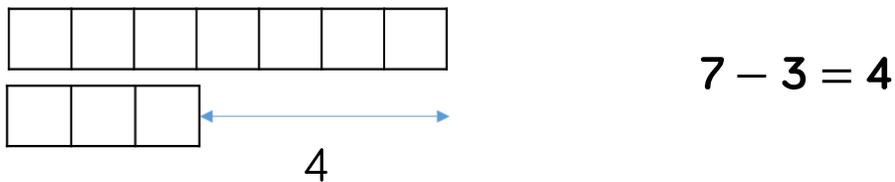
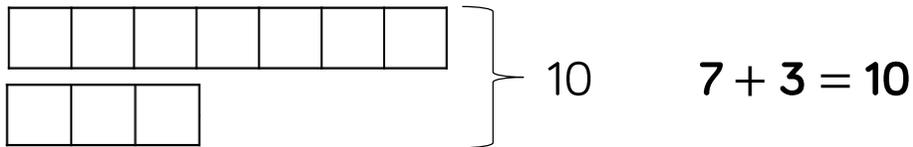
The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

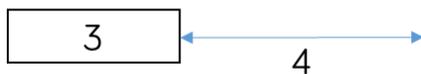
In KS2, children can use bar models to represent larger numbers, decimals and fractions.

Bar Model (multiple)

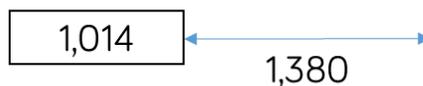
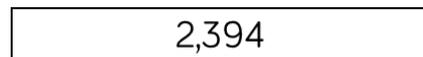
Discrete



Continuous



$$7 - 3 = 4$$



$$2,394 - 1,014 = 1,380$$

Benefits

The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

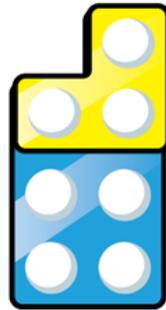
Number Shapes



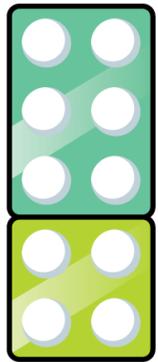
$$7 = 4 + 3$$



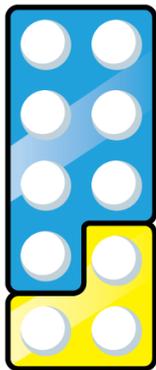
$$7 = 3 + 4$$



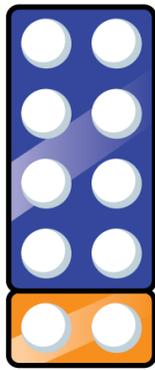
$$7 - 3 = 4$$



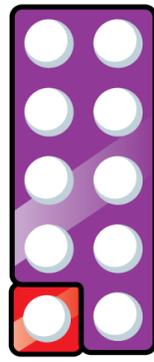
$$6 + 4$$



$$7 + 3$$



$$8 + 2$$



$$9 + 1$$

Benefits

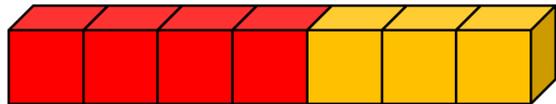
Number shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

When adding numbers, children can see how the parts come together making a whole. As children use number shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.

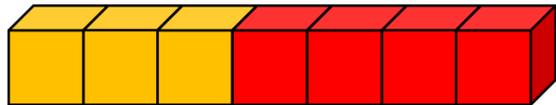
When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing. Again, children will start to be able to subitise the part that is missing due to their familiarity with the shapes.

Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.

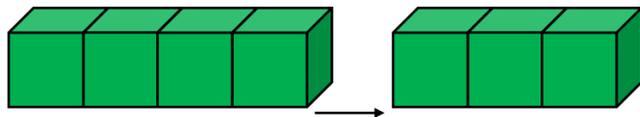
Cubes



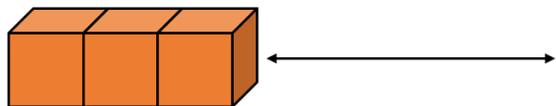
$$7 = 4 + 3$$



$$7 = 3 + 4$$



$$7 - 3 = 4$$



$$7 - 3 = 4$$

Benefits

Cubes can be useful to support children with the addition and subtraction of one-digit numbers.

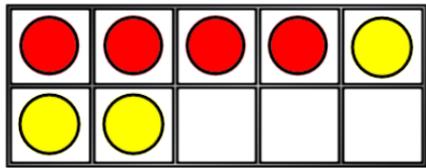
When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.

When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers.

Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.

Ten Frames (within 10)



$$4 + 3 = 7$$

$$3 + 4 = 7$$

$$7 - 3 = 4$$

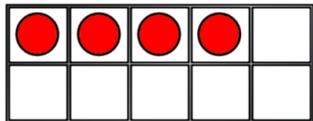
$$7 - 4 = 3$$

4 is a part.

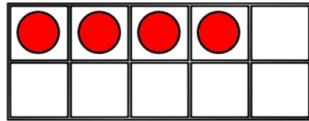
3 is a part.

7 is the whole.

First

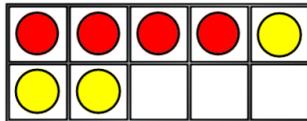


Then

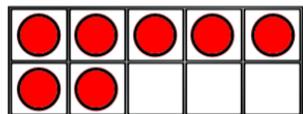


$$4 + 3 = 7$$

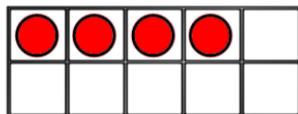
Now



First

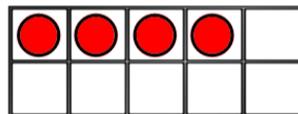


Then



$$7 - 3 = 4$$

Now



Benefits

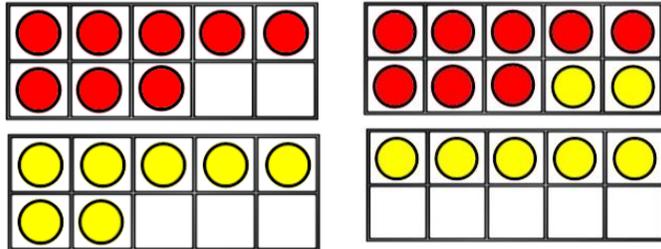
When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.

Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts. Using these structures, the ten frame can enable children to find all the number bonds for a number.

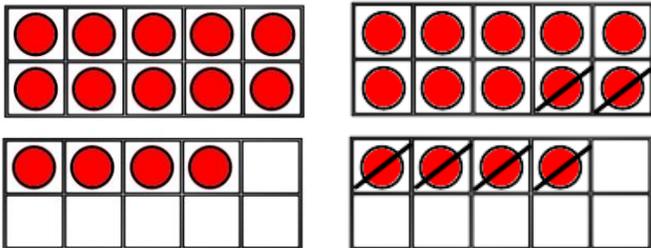
Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

Ten Frames (within 20)



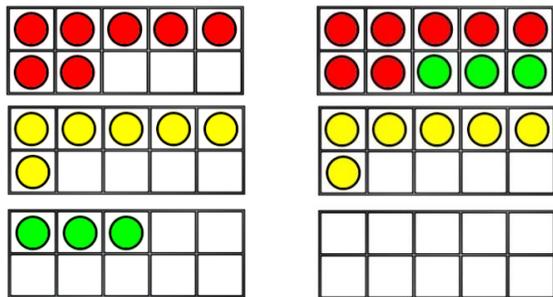
$$8 + 7 = 15$$

Diagram showing 8 partitioned into 2 and 5. A blue oval encircles the 8 and the 2.



$$14 - 6 = 8$$

Diagram showing 14 partitioned into 4 and 2. A blue oval encircles the 14 and the 4.



$$7 + 6 + 3 = 16$$

Diagram showing 10 partitioned into 7 and 3. Lines connect 10 to 7 and 3.

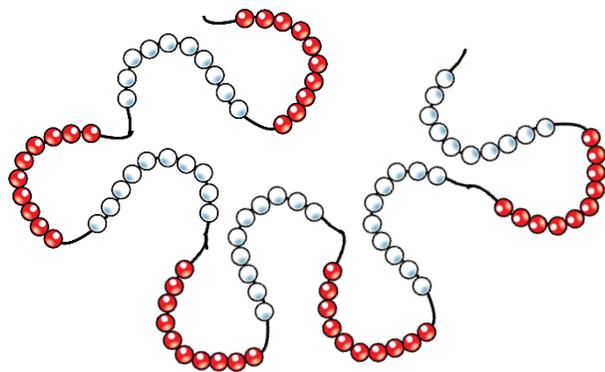
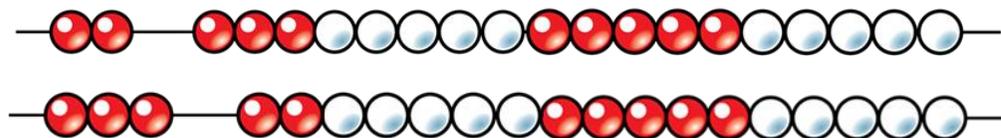
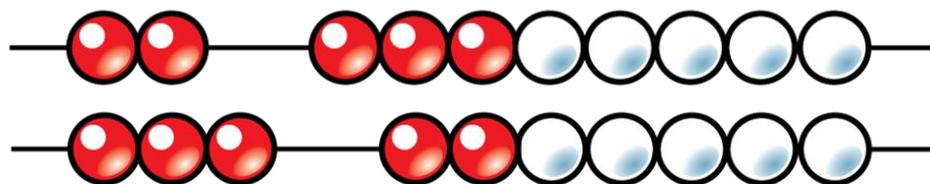
Benefits

When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition.

When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.

When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.

Bead Strings



Benefits

Different sizes of bead strings can support children at different stages of addition and subtraction.

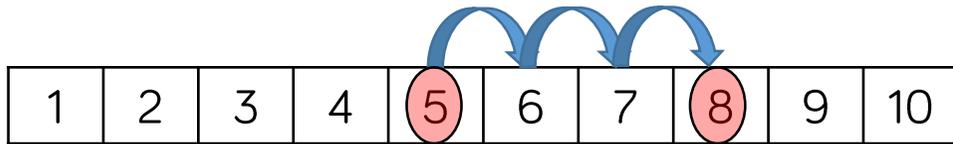
Bead strings to 10 are very effective at helping children to investigate number bonds up to 10. They can help children to systematically find all the number bonds to 10 by moving one bead at a time to see the different numbers they have partitioned the 10 beads into e.g. $2 + 8 = 10$, move one bead, $3 + 7 = 10$.

Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20.

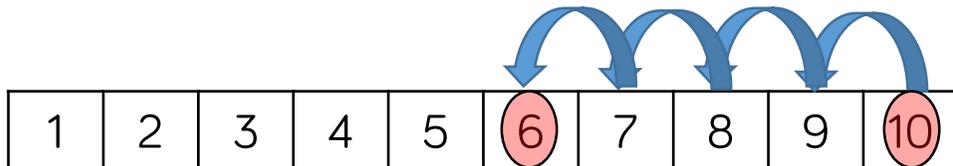
Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making ten. Bead strings can show a link to adding to the next 10 on number lines which supports a mental method of addition.

Number Tracks

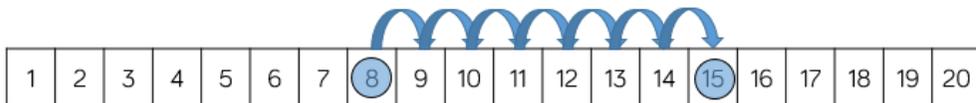
$$5 + 3 = 8$$



$$10 - 4 = 6$$



$$8 + 7 = 15$$



Benefits

Number tracks are useful to support children in their understanding of augmentation and reduction.

When adding, children count on to find the total of the numbers. On a number track, children can place a counter on the starting number and then count on to find the total.

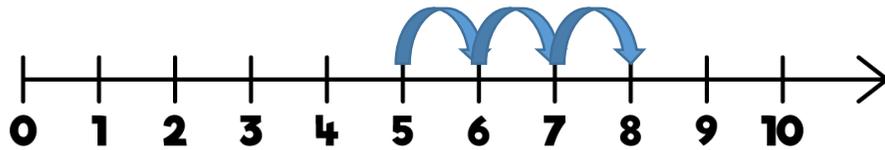
When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers.

Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.

Playing board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.

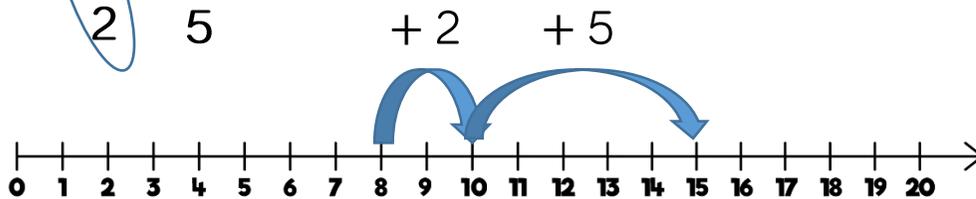
Number Lines (labelled)

$$5 + 3 = 8$$



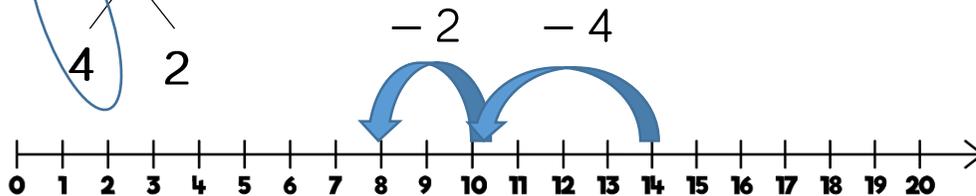
$$8 + 7 = 15$$

2 5



$$14 - 6 = 8$$

4 2



Benefits

Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

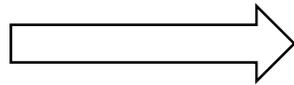
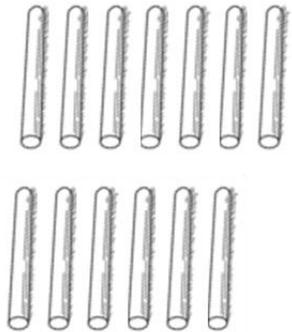
Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

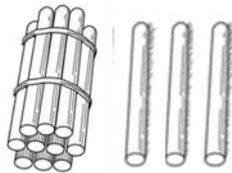
Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

Straws

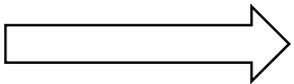
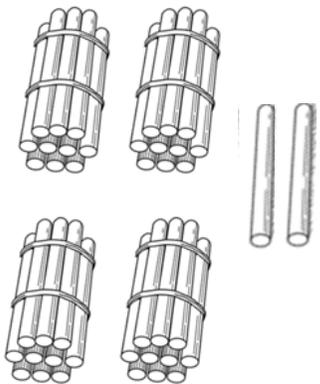
$$7 + 6 = 13$$



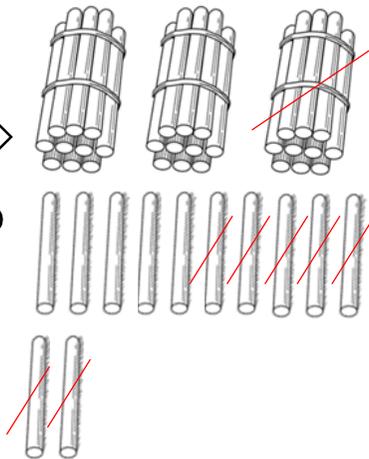
bundle together
groups of 10



$$42 - 17 = 25$$



unbundle group
of 10 straws



Benefits

Straws are an effective way to support children in their understanding of exchange when adding and subtracting 2-digit numbers.

Children can be introduced to the idea of bundling groups of ten when adding smaller numbers and when representing 2-digit numbers. Use elastic bands or other ties to make bundles of ten straws.

When adding numbers, children bundle a group of 10 straws to represent the exchange from 10 ones to 1 ten. They then add the individual straws (ones) and bundles of straws (tens) to find the total.

When subtracting numbers, children unbundle a group of 10 straws to represent the exchange from 1 ten to 10 ones.

Straws provide a good stepping stone to adding and subtracting with Base 10/Dienes.